Name: _____

Group _____

1) Using the following density function

$$f_X(x) = \begin{cases} \frac{k}{x^4} & x \ge 1\\ 0 & else \end{cases}$$

a) What is the constant k that makes the following function a valid density?

b) Find P(X = 4)

c) Find P(-0.5 \leq X \leq 5)

d) Find P(X > 3)

e) Find the cdf.

f) Using the cdf, calculate the density.

g) Calculate the 25th percentile of X

h) Find the expected value of X.

i) Find the variance of X.

2) Using the following joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{44}(x+y)^2 & 0 \le x \le 2, 0 \le y \le 3\\ 0 & else \end{cases}$$

a) What is the probability that Y > 2X?

b) Determine the CDF.

- c) Determine the density from the CDF.
- d) Find the marginal density of X, $f_X(x)$.
- e) Are X and Y independent? Why?
- f) Calculate the conditional density of Y when X = x where $0 \le x \le 2$.
- g) What is the conditional probability that Y is between 0.5 and 1.5 given that x = 1.
- h) What is the expected value of XY?

4). For the following questions, indicate which can be true. If an answer is false, state why it is false.

a) A CDF $F_X(x)$ can have a value of 4.3.

b) A density $f_X(x)$ can have a value of 4.3.

c) A mass $p_X(x)$ can have a value of 4.3.

d) A CDF $F_X(x)$ can be 1 for two or more values of x.

e) A density $f_X(x)$ can be 1 for two or more values of x.

f) A mass $p_X(x)$ can be 1 for two or more values of x.

g) The area under the $F_X(x)$ curve from - ∞ to ∞ is 1.

h) The area under the $f_X(x)$ curve from - ∞ to ∞ is 1.

i) The CDF $F_X(x)$ can be negative.

j) The density $f_X(x)$ can be negative.

k) The mass $p_X(x)$ can be negative.

I) The graph of $f_X(x)$ can have jumps (i.e., can be discontinuous).

m) The graph of $F_X(x)$ for a discrete random variable can have jumps.

n) The graph of $F_X(x)$ for a continuous random variable can have jumps.

o) The graph of $p_X(x)$ can have jumps.